Core IX

Complex Analysis-I

Course Objectives:

The objective of the course is to introduce the theories for functions of a complex variable. The concepts of analyticity and complex integration and its applications, are discuss in detail. This course is prerequisite to many other advance analysis courses such as advanced complex analysis, geometric functions, theory, potential theory, theory of entire and meromorphic functions, etc.

Learning Outcomes:

After completing the course the student will be able to

- Understand the geometric aspects of complex numbers system, convergence of series of complex numbers.
- Understand the significance of complex differentiability, analyticity and construction of analytic functions from given harmonic functions.
- Relate the notion of line integral, Cauchy fundamental theorems on integrals and its applications.
- Classify the nature of singularities, properties of zeros and poles and be able to know the applications of residue theorem.

Unit I

Basic properties of complex number and, Stereographic projection, power series, absolute convergence, uniform convergence, Cauchy-Hadamard formula for the radius of convergence, circle of convergence, exponential, logarithmic, sine and cosine functions for complex numbers.

Unit II

Continuity and differentiability of a complex valued function, analytic function, necessary and sufficient conditions for analytic functions, Cauchy-Riemann equations (Cartesian and polar form), harmonic and conjugate harmonic functions, construction of analytic function (Milne-Thomson's method).

Unit III

Line integral, path independence, complex integration, Green's theorem, anti-derivative

theorem, Cauchy-Goursat theorem, Cauchy integral formula, Cauchy's inequality, derivative of analytic function and its generalizations, Liouville's theorem, Morera's theorem, Taylor's and Laurent's theorem, expansion of analytical function in Taylor and Laurent series.

Unit IV

Zeroes of an analytic function, singularities of complex functions and its classifications, residues, Cauchy's residue theorem, residue at infinity, residues at poles and its examples, maximum modulus theorem.

Books Recommended:

- ✓ *Elias M. Stein & Rami Shakarchi: Complex Analysis, Princeton University press, Princeton and Oxford, 2003.*
- ✓ *Joseph Bak and Donald J. Newman: Complex analysis (3rd Edition), Undergraduate Texts in Mathematics, Springer-Verlag, NewYork, 1997.*

Books for Reference:

- ✓ *S. Ponnusamy and Herb Silverman: Complex variables with Applications: Birkhauser, (2006) (Indian Edition 2012).*
- ✓ *H. A. Priestly: Introduction to Complex Analysis, Oxford University Press, 2008.*
- ✓ *Donald Sarason: Complex Function Theory: AMS, Second Edition, 2007.*
- ✓ *James Ward Brown and Ruel V.Churchill: Complex Variables and Applications (Eighth Edition), McGraw-Hill International Edition, 2009.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs*
- ✓ *e-Learning Source http://ndl.iitkgp.ac.in ; http://ocw.mit.edu ; http://mathforum.org*