

## **Core IV**

### **Algebra-I**

#### **Course Objectives:**

To present a systematic and rigorous study on algebraic structures like groups, rings and some important results with their applications. After pursuing this course, one can opt for advanced topics in groups, rings and their applications to problems in physics, computer science and engineering.

#### **Learning Outcomes:**

After completing this course, students will be able to

- Understand permutation groups with some results and application in Rubik's cube.
- Understand the concept of homomorphisms, isomorphisms, normal subgroups and factor groups.
- Explore more properties of rings and ideals rigorously.
- Get introduced to the concept of reducibility and irreducibility of polynomials and concept of unique factorization domain.

#### **Unit I**

Permutation groups, definition and notations, cyclic notation, properties of permutations, isomorphisms, definition and examples, Cayley's theorem, properties of isomorphisms, automorphisms, cosets, properties of cosets, Lagrange's theorem and consequences, an application of cosets to permutation groups, an application of cosets to Rubik's cube.

#### **Unit II**

External direct products, definition and examples, properties of external direct products, the group of units modulo  $n$  as an external direct product, applications, normal subgroups, factor groups, application of factor groups, internal direct products, group homomorphisms, definition and examples, properties of homomorphisms, the first isomorphism theorem.

#### **Unit III**

Characteristic of a ring, ideals, factor rings, prime ideals and maximal ideals, ring homomorphisms, definition and examples, the field of quotients, polynomial rings, notations and terminology, division algorithm and consequences.

## Unit IV

Factorization of polynomials, reducibility test, irreducibility test, unique factorization in  $\mathbb{Z}[x]$ , divisibility in integral domains, irreducible, primes, unique factorization domain, Euclidean domain.

### Books Recommended:

- ✓ *Joseph A. Gallian, Contemporary Abstract Algebra (9th Edition), Narosa Publishing House, New Delhi, 2010.*
- ✓ *I. N. Herstein, Topics in Algebra, Wiley Eastern Limited, India, 1975.*

### Books for Reference:

- ✓ *John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.*
- ✓ *D. S. Dummit, R. M. Foote, Abstract Algebra, Wiley-India edition, 2013.*
- ✓ *Joseph I. Rotman, An Introduction to the Theory of Groups, 4th Ed., Springer Verlag, 1995.*
- ✓ *M. Artin, Abstract Algebra, 2nd Ed., Pearson, 2011.*
- ✓ *Suggested digital platform: NPTEL/SWAYAM/MOOCs.*
- ✓ *e-Learning Source <http://ndl.iitkgp.ac.in> ; <http://ocw.mit.edu> ; <http://mathforum.org>*